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ABSTRACT

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Research Report 88-4

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Pamela Schram, Sandra Wilcox,
Perry Lanier, and Glenda Lappan

**National Center for Research on Teacher Education**

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Abstract

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This study examines the piloting of a sequence of innovative mathematics courses for undergraduate education majors. These courses emphasize the conceptual foundations of mathematics and actively engage prospective elementary teachers in making sense of mathematical situations. The main question informing the study is, What is the nature and extent of changes in the knowledge about mathematics, mathematics learning, and mathematics teaching among students as a result of these courses?

Results from this study suggest that change in two important areas occurred in student thinking about mathematics as a consequence of this intervention: a change in students' conception of what mathematics is and a change in their perception of what a mathematics class is like and in their knowledge of how mathematics is learned.

# CHANGING MATHEMATICAL CONCEPTIONS OF PRESERVICE TEACHERS: A CONTENT AND PEDAGOGICAL INTERVENTION[1]

Pamela Schram, Sandra Wilcox,
Perry Lanier, and Glenda Lappan[2]

## Reconsidering the Traditional Approach to Mathematics

Mathematics is perhaps the weakest area of the elementary curriculum. Since the late 1950s, mathematics education has been under continuous scrutiny. The slogans characterizing reform agendas are all too familiar--new math, back-to-basics, the new basics, computer literacy, and problem solving. Public declarations regarding the aims of mathematics education may have changed but the profile of a typical math class has remained virtually untouched by the rhetoric. Mathematics instruction begins with checking the previous day's assignment. Troublesome problems are worked by the teacher or a student. Then the teacher briefly explains the next piece of material and the remainder of the time is spent at seatwork on the next assignment (Conference Board of Mathematical Sciences, 1975; Davis and Hersh, 1981; Peterson, 1988; Stodolsky, 1987; Welch, 1978).

Despite the efforts to reform mathematics education, children are not learning much mathematics and what they are learning is of questionable value (Reys, Suydam, and Lindquist, 1984). It is estimated that elementary children spend up to 90 percent of math instruction on paper-and-pencil practice at computation skills learned by rote (Burns, 1986). They are taught procedures and algorithms to manipulate numbers and symbols without understanding the

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[1]This paper was presented at the annual meeting of the American Educational Research Association in New Orleans in April 1988.

[2]Pamela Schram and Sandra Wilcox are researchers for the National Center for Research on Teacher Education and instructors in teacher education at Michigan State University. Perry Lanier and Glenda Lappan are senior researchers with the NCRTE; Lanier is a professor in the Department of Teacher Education and Lappan is a professor in the Department of Mathematics at MSU.

meaning of symbolic representations or the meaning of mathematical processes. As a consequence, children come to view mathematics as a collection of facts and senseless rules to be memorized and filed for future reference. Data from the last 20 years show the impoverished results of this computational focus (Erlwanger, 1973; McKnight et al., 1987; National Assessment of Educational Progress, 1983; Schoenfeld, 1985).

A number of factors contribute to this state of affairs including assumptions about what it means to know mathematics, how children learn mathematics, and what are effective ways of teaching mathematics. An instrumental, technical orientation informs this traditional approach to teaching and learning mathematics. Learners are seen as passive receptacles into which mathematical knowledge is poured. Teachers are viewed as mere technicians who implement curriculum conceived by "experts." Mathematics is assumed to be static, rule-bound, and linearly ordered. Instruction is presumed to be most effectively and efficiently organized for mastery learning by breaking content into small pieces to be digested. Learning is assessed through paper-and-pencil tests where being able to select the correct answer is taken to <u>be</u> knowledge.

A decade of research and deliberation in mathematics and mathematics education has produced findings that challenge this traditional orientation to the teaching and learning of mathematics. At a formal level, mathematics is the systematic study of magnitude, relations between figures and forms, and relations between quantities expressed symbolically. More informally, it is the dynamic, everyday human activity of analyzing and describing the numerical and spatial aspects of our world. The discipline of mathematics is growing and changing. Over half of all mathematics has been invented since World War II (Davis and Hersh, 1981). It is impossible for any one person to know all

there is to know or to be able to predict the specific mathematical content of problems that one might encounter. Seeing mathematics as a dynamic human activity means that one values *doing* mathematics over accumulating facts *about* mathematics. Mathematics instruction that places emphasis on the absorption of the "record of knowledge" (Dewey, 1904/1964; Romberg, 1983) is no longer appropriate. The goal should not be the accumulation of large numbers of problems with appropriate algorithmic solutions but to learn ways of making sense of mathematics, inventing procedures to solve new problems, and building models to understand mathematical situations.

Recent research on concept acquisition is adding to our knowledge of how children think about numbers, geometric concepts, and relationships among variables (Carpenter, Moser, and Romberg, 1982; Ginsberg, 1977; Resnick, 1983). Children are not passive learners but actively construct, interpret, and put structure on new mathematical learning (Resnick and Ford, 1981; Romberg and Carpenter, 1986). Children come to school with a rich informal knowledge of mathematics and demonstrate a natural capacity for and interest in understanding mathematical concepts. Traditional mathematics instruction, however, is not conducive to the development of an inquiry orientation to learning and doing mathematics. A change is required in the way mathematics is organized and taught and in teacher beliefs about what it means to know and do mathematics. This includes a different perspective on student and teacher role during instruction and a shift in orientation from a computational to a conceptual focus.

Leaders in mathematics education--the National Council of Teachers of Mathematics, the National Science Board, the Board of Mathematical Sciences, and the American Association for the Advancement of Science--are calling for a reorganization of the mathematics curriculum around concept development and

problem solving. They argue that mathematics is a creative, everyday human activity that cannot be built exclusively on rules and routines. Their recommendations to teachers include not only reducing the amount of time devoted to pencil-and-paper drill-and-practice on computational skills but also engaging children in challenging problem situations even though they have not completely mastered computational skills; providing problem situations in forms other than traditional textbook word problems; creating a classroom where questioning, exploration, reasoning, and justification are encouraged and expected; and using the power of computing technology to free students from tedious computations and to allow them to concentrate on problem-solving processes (National Council of Teachers of Mathematics, 1980). In addition, several state departments of education (e.g., California, Texas, Wisconsin, and Oregon) are recasting curriculum goals and textbook adoption guidelines that support these recommendations.

Reorganizing mathematics curriculum and instruction around concept development and problem solving poses several substantial problems. Implementation of a conceptually based, problem-solving approach to mathematics instruction requires teachers to have a conceptual understanding of mathematics, to know why understanding concepts is important, and to know how to help students gain that understanding (Devaney, 1983; Lampert, 1986; Resnick, 1983; Shulman, 1986). They need to be able to comprehend how various mathematical concepts relate to the larger field of mathematics (Steinberg, Haymore, and Marks, 1985). For many elementary teachers, the limitations of their knowledge about mathematics and teaching mathematics constrains their ability to teach conceptually. These limitations originate in their own experience, as learners of mathematics and as students in elementary teacher preparation programs. If prospective elementary teachers are to overcome

these limitations, they must have opportunities in their teacher preparation programs to deepen their knowledge about the nature of mathematics, children's mathematics learning, and instructional practices that promote conceptual understanding.

## A Study of Innovative Math Courses

This study examines the piloting of a sequence of innovative mathematics courses for undergraduate education majors.[3] These courses emphasize the conceptual foundations of mathematics and actively engage prospective elementary teachers in making sense of mathematical situations. The environment is constructed in such a way that these prospective teachers experience mathematics much as their own students might. The instructors take the point of view that students should experience mathematics as a dynamic engagement in solving problems that arise both within mathematics and outside of mathematics.[4] The main question informing the study is, What is the nature and extent of changes in the types of knowledge about mathematics, mathematics learning, and mathematics teaching among students as a result of these courses? This study addresses the discrepancy between (a) conceptions of mathematics, mathematics learning, and mathematics teaching derived from research and deliberation in mathematics, mathematics education, cognitive

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[3]The project reported here is a within-site study that is part of the Teacher Education and Learning to Teach Study being conducted by the National Center for Research on Teacher Education.

[4]The selection of mathematical ideas to pursue in the courses came from consideration of the following questions: Is it good mathematics? Is it important? What does knowing this idea enable a student to do? To what is it connected? How does it relate to the big mathematical ideas for elementary/middle school children? How does the content selected represent mathematics to prospective teachers? Does the context require students to engage in doing mathematics--analyzing, inventing, proving, and applying?

science, and educational psychology, and (b) how mathematics is commonly conceived, taught, and learned in most K-12 and university mathematics courses taken by prospective elementary teachers.

Part of a larger longitudinal effort to study both knowledge and contextual constraints in implementing a conceptual approach to mathematics in elementary classrooms, the study reported here investigates how the intervention of this mathematics sequence influences the responses teacher candidates initially bring to questions such as, What is mathematics and what does it mean to know mathematics? What should children study and what are guiding principles for selecting curriculum in elementary school mathematics? How is mathematics learned and what are effective ways of building mathematical experiences for children? What is the teacher's role and what does she/he need to know to teach elementary mathematics? What are mathematics classes like and what causes them to be this way?

Data Source/Method

The subjects of this study are students at Michigan State University, enrolled in the first course of an innovative, conceptually based, three-course mathematics sequence for prospective elementary teachers.[5] The courses take an overall integrated approach to mathematics but each course has a major emphasis allowing an in-depth probe of ideas. The content of the first course

[5]Perry Lanier directs the project, Glenda Lappan is associate director of the project and principal designer and instructor for the sequence of mathematics courses. Ruhama Even, a doctoral candidate in math education, assisted in course development.

is devoted to an exploration of numbers and number theory and emphasizes patterns, relationships, and multiple representations of problem situations.[6]

During the 10-week course, students were observed during 30 hours of instruction. Data for the study include recorded fieldnotes of classroom observations. Students in the course completed questionnaires at the beginning and at the conclusion of the course. Six students were selected to participate in tape-recorded interviews at three points during the course. Researchers collected copies of notes, assignments, and tests from these six students. All data were subjected to qualitative analysis procedures (Bogdan and Biklin, 1982) and analyzed for instances of common phenomena. Triagulation of data gathered at different points in time, in different contexts, and from different subjects was used to check the validity of inferences (Hammersley and Atkinson, 1983). As analysis of the data suggested constructs or frameworks researchers looked for instances of confirming and disconfirming evidence.

## Establishing an Environment for Learning

It was clear on the first day of class that this mathematics course would not follow the typical routine of math instruction. The instructor described the following scenario that she called the "locker problem":

> In a certain high school there were 1000 students and 1000 lockers. Each year for homecoming the students lined up in alphabetical order and performed the following ritual: The first student opened every locker. The second student went to every second locker and closed it. The third student went to every third locker and changed it (i.e., if the locker was open, he closed it; if it was closed, he opened it). In a similar manner, the fourth, fifth, sixth, . . . student changed every fourth, fifth, sixth, . . . locker. After all 1000 students had passed by the lockers, which lockers were open?

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[6]The emphasis of the second course is geometry; the third course is data analysis and interpretation. In each term connections among number, geometry, probability, and statistics are made.

The instructor asked students to brainstorm together, "How would you tackle a problem this big?" Students identified two possible strategies. One student suggested making a diagram of a simpler problem and another student made a conjecture concerning prime numbers. The instructor suggested they form small groups and explore possible solutions. From the first day, she was creating an environment where students could collectively and cooperatively learn ways to make sense of mathematics, invent procedures to solve new problems, and build models to understand mathematical situations.

The "routine" of this class stood in sharp contrast to the routine of traditional mathematics classes. Students worked in groups to explore, conjecture, and validate possible solutions for the problem situation. They used multiple representations to examine parallel relationships. Students identified connections among mathematical ideas. The students were encouraged to generalize their solutions and communicate results from their explorations of mathematical ideas. The painted cube problem is illustrative.

Students were given a single cube.

| | |
|---|---|
| Instructor: | What makes this cube a cube? Talk to me about a cube. What are its invariant properties? |
| Arlene:[7] | It has six sides. |
| Instructor: | What do you mean by a side? |
| Arlene: | I mean it has six faces. |
| Kristin: | All faces are equal in surface area and length of edges. |
| Instructor: | What are some other properties of this cube? Close your eyes and let your hand roam over this cube. |
| Chris: | It has eight corners. |

[7]All names of students are pseudonyms.

Students offered other properties. The instructor described a single cube as a cube on its first birthday. She then asked students to build a cube on its second birthday. Students built and described a cube on its second and third birthdays.

Instructor: We have magic glue that will hold together a 10-year-old cube. This magic glue will hold long enough to enable us to pick up the cube and dunk it in black paint. When the 10-year-old painted cube is removed from the paint, the glue releases its hold on the cubes. How many cubes will we have? [There was considerable discussion to resolve that question.] We have a pile of 1000 cubes. What do you think I am going to ask you?

Arlene: How many cubes are painted black on one face, two faces, three faces?

There was further discussion to make sure that everyone was clear about the nature of the problem. Then the instructor said, "Make a prediction for an *n* year-old-cube." Students divided themselves into small groups and gathered data, organized the data into tables, and looked for patterns related to cubes aged 2 through 10. Students worked in their groups building cubes, of various ages and debated possible patterns that were emerging. Students were so engrossed in this problem that they continued to work past the time class would usually end.

The environment of the classes was created to provide opportunities for students to encounter mathematics in a different way. Problem situations such as those described were used consistently to introduce mathematical concepts. These problems were selected to engage students in mathematical inquiry: to explore the richness of the embedded mathematics; to discover patterns and relationships among mathematical ideas. In the context of small groups, students were encouraged to make conjectures, validate assertions with convincing arguments, and communicate with others as they attempted to make

sense of mathematical situations. In the next section we describe changes in two aspects of student thinking about teaching and learning as a result of this course.

## Description and Interpretation of Findings

Results from this study suggest that change in two important areas occurred in student thinking about mathematics and about mathematics teaching and learning as a consequence of this intervention: a change in students' conception of what mathematics is and a change in their perception of what a mathematics class is like and in their knowledge of how mathematics is learned. The changes were most evident in relation to how they thought about math as adult learners but less so in relation to children as learners.

### A Change in Conception of What Mathematics Is

Traditional mathematics instruction promotes an image of mathematics as an abstract, mechanical, and meaningless series of symbols and rules. Mathematics is treated as isolated bits and pieces packaged as "today's lesson" and focused on a single skill or topic. The fragmentation of mathematical concepts does not provide opportunities for students to understand how these pieces fit together in the larger field of mathematics. This traditional view of mathematics limits one's ability to confront a new problem. The only strategy available is to search one's memory for the answer (fact) or the algorithm (procedure) for getting the answer. This places increasing stress on learners proceeding through school mathematics to accumulate huge numbers of problems and algorithms. Such a view of mathematics does not provide one with tools to mathematize situations encountered in daily living.

Students in our study, not surprisingly, entered this course with a traditional view of mathematics. During the first interview we provided an opportunity for students to talk about their experiences with mathematics and what it means to know mathematics. One student, Jayne, talked about her previous math experiences and compared them to the expectations for this class:

> Up till then [the university math placement exam] I just plugged the numbers in and I always got good grades. It had been a long time since I had any math. I couldn't remember the way to do lots of the problems or appropriate formulas [to solve them]. I had no ability to tackle them if I didn't know the formula. . . . I like to plug numbers into formulas. This [math class] is very up-setting. This is the first time I ever thought about "why." In high school algebra we just plugged in the numbers, just waited for the formulas. I realize I am going to have to learn to think about it [why] if I expect to teach math.

A week earlier during a class discussion of different strategies students used to solve the locker problem, Jayne expressed her frustration: "I was upset that I couldn't come up with a formula for it."

By the end of the course, the majority of the students were beginning to question this traditional notion of what it means to know mathematics.

> Christine: In my earlier math classes, it has always been *one and only one way* to solve a problem. Math 201 has shown me many new ways to approach a problem. More important is the fact that 201 has shown me *why* a certain procedure is used and *why* it works. There is more to math than questions and answers.
>
> Arlene: The class made me realize that there are many mathematical rules and procedures we use without understanding why they work. I gained a deeper sense of the foundations of math [number structures].
>
> Melanie: Before [this class] math was a big mystery. Now I feel I've gotten more behind the lines. I'm more aware what's going on. I feel I can actually *see* the patterns instead of being told that they're there.
>
> Kelly: I feel I have a better understanding of numbers, patterns, and math overall. . . . I'm thinking of math as more than memorization of principles and concepts.

Kelly saw a logical ordering to the concepts covered in the course. She described "patterns, sequences, change, and relationships between changing things" as a common thread. "This whole class has dealt with patterns through shapes, story problems, number patterns and now we're seeing it based on similarity and congruency." She saw how certain patterns are encountered "over and over," first by exploring numbers and then incorporating shapes and figures. She gave triangular numbers as an example and said now she "looks for them."

In one written assignment students were asked to reflect on an article describing the benefits of understanding mathematics problems at a conceptual level. More than half the students responded by making connections to this course and their own personal experiences:

> In math it's necessary that the student learn the theory behind a concept because so much of math is interrelated. By learning the theory behind solving a problem rather than by simply solving it, students can use that knowledge and apply it to other situations. If you simply do problems without really thinking about why a concept works, it will mean little to you as soon as you hand in your assignment. An example of this method of learning is how we deal with problems in our class. Take for example the problems with building steps out of rods. Given a certain situation you may be able to figure out the answer. But we never leave it at that. We always think of equations for solving for *n* number of rods (or whatever the problem may be dealing with). In this way we are able to use our knowledge to solve other similar situations. The knowledge just doesn't apply to a single problem. And that's the whole point in learning math--not just learning a concept, but learning how to use it. (Chris)

> I learned the formula (l x w)2 + (w x h)2 + (l x h)2 = the surface area of a rectangular prism and the formula (s)6 = the surface area of a cube a *long* time ago. I had no idea what that meant, simply that by substituting numbers for the letters I could get the "right" answer. I managed to forget the formulas as soon as I left high school. I have been sewing, without patterns for many years--seems unrelated, but. . . . In class we were asked to make a "jacket" for a rectangular prism composed of twelve cubes. As I started counting the units on the jacket, I knew there had to be pattern in this somewhere. So I made another 12-cube rectangular prism, and a jacket for it. Then I counted the units in both jackets and began looking for a pattern. After thinking about other sizes of

> rectangular prisms I arrived at the method [formula] that would work for any rectangular prism, including a cube and suddenly the light dawned and I realized I had been using this idea all along, in my sewing. Now I feel that I do have a solid understanding of surface area, I can say I "own" this concept--and I can even explain it to someone else. (Jayne)

As a result of this course the majority of the students were questioning the traditional view of mathematics they brought to the course. They no longer were satisfied *in their own learning* with just searching for the right formula or algorithm. They were beginning to appreciate the value of a conceptual understanding of the many facts they had accumulated as learners of mathematics.

However, the traditional notions they brought to the course about mathematics at the elementary level were not significantly challenged. In the final interview we asked the students to talk about elementary mathematics. Nearly half the students still associated elementary mathematics with basics--number facts and whole-number operations.

> Chris: In lower elementary it's important to get down the basics--the four operations, addition, subtraction, multiplication, and division. Those are things you have to learn before you can really do a lot else. At sixth and seventh grades they have a pretty good knowledge of the basics and you can start with algebra and stuff, more on problem solving and applications. The way we've learned can be emphasized more after they've got the basics down. In early elementary you have more memorizing. . . . Once you've got that learned you can move on to the way that we've learned with the groups working and talking about the problems.

Data from the final questionnaire suggest that more than half the students still saw elementary mathematics as hierarchically ordered--that computational skills must be mastered before problem solving, and that skills at one level must be mastered before proceeding to the "next" level.

It appeared that this course had been a powerful intervention in how they were thinking about mathematics for themselves. It provided conceptual

understanding for many facts, formulas, and rules that they had accumulated and committed to memory. While they saw the benefits of understanding mathematics at a conceptual level for themselves that did not seem to carry over to how they thought about mathematics for young children.

A Change in Knowledge of How Mathematics Is Learned

Prospective elementary teachers have logged many hours in traditional mathematics instruction, the focus of which has been largely computational and procedural. Their courses at the university have reinforced a notion that learning mathematics means attending classes, taking notes during the instructor's lecture, completing assignments, memorizing procedures and formulas, and passing tests. Opportunities to experience alternative ways to do and learn mathematics may contribute to changes in their knowledge of how mathematics is learned and effective ways to build mathematical experiences for learners.

Students in this course entered expecting to learn through teacher lecture and demonstration. What they encountered led them to an appreciation (a) the use of problem situations to explore mathematical ideas, (b) for opportunities for group work to investigate problem situations, and (c) opportunities to talk with others in the class about the mathematics they were learning.

Problem situations to explore mathematical ideas. Each new mathematical concept was introduced with a problem situation. To begin an exploration of factors and multiples, students were given the locker problem at the first class meeting. In the initial interviews, we asked students what they thought about beginning the class this way. All the students interviewed expressed surprise at being able to make sense of a problem that initially seemed beyond their ability to solve.

Maria: Oh no, story problems! But then it made sense, I liked it. Probably the first time I ever understood a story problem. Maybe because it was just factors I could figure it out, not that complex. But in class we had to relate it to prime numbers.

Kelly: I thought it was going to take forever to figure it out. We worked there [in class] up to one certain number--I think 20.

Interviewer: What did you do on your own at home?

Kelly: I went through 20. Based on that I looked at the sequence and found a lot more closed than open. Then I discovered the ones open were the ones with direct square roots. I knew that factors would determine open and close, but I had no idea until I saw the sequence about direct square roots.

Interviewer: What question came to mind when you saw 1, then 4, then 9?

Kelly: Before that I had an idea that numbers with an even number of factors would be closed.

Kristin: I thought, *Wow*! How do you go about doing this? I enjoyed working on it. I liked the group better, listening to how they would go about it. I'm getting confidence. I said to my husband, "I actually *did* this and did it right!"

In subsequent interviews, students responded to a question about what purposes they thought these problem situations served:

Maria: Mainly they help us think how to get to an answer rather than sticking in an equation and seeing if an answer comes out. Class always carries over. We start small. We talked about one cube, then we kept building until we finally got to an equation of the whole thing . . . filling out a chart. That's really important. You see a pattern. *Gosh*, I didn't even realize that!! [said with surprise at the connection she had just made and pausing to think about that for a moment]. Instead of giving us a formula, we figure it out for ourselves. There are a lot of patterns we do use. It [looking for patterns] works, believe me! I use it on a test. I think I understand math better.

Kristin: I've thought about math in a different way. All this problem-solving kind of thing we've been doing really made me think differently. Instead of looking and thinking "Oh, I can't do that," I think there's gotta be a way so I start thinking about different ways like

working backwards. You're more involved when looking and doing . . . Now at least I feel like I can try to do something.

Two students contrasted these problem situations with the assignments of a typical math class:

Melanie: I still get frustrated a lot but I am more satisfied when I can figure things out rather than just doing busy work. I used to think math was just busy work. This class doesn't have any busy work.

Interviewer: Can you give me an example of busy work?

Melanie: Do 30 problems on the distributive property when they're just different numbers, all the same thing, mindless by the 30th problem, like practice work.

Chris: [talking about a typical math class] Lots of homework, constantly turning in assignments. Homework every single night. Here's 30 problems, do them. Turn them in tomorrow. Here's 30 more problems! This class is a lot better.

A consistent theme, the appearance of patterns, punctuated nearly all student questionnaire comments related to problem solving:

A study of patterns is important because patterns are evident in many aspects of math. Once you can see a pattern, you want to continue trying to find others. Patterns serve to bring many ideas together and to relate math from one area to another. (Kristin)

They're constantly popping up in math--all sorts of sequences like triangular and Fibonacci numbers. (Chris)

Repetitions and patterns are frequently found in math problems. By finding a recurring pattern, one can get a sense of what is occurring in a problem and then know how to approach it. (Mary)

Some of the problems involved number patterns, some were shape and figure patterns, others were patterns by increasing size. . . . These problems are dealing with ideas and we're putting them together for a solution. I think for a while. What is the specific change? How is this change related to that change? They're trying to get us to think logically, to find a specific pattern, set up specific steps in each problem that ends in a generalization. (Kelly)

Three of the six students interviewed were beginning to consider the value of problem situations for their future teaching:

> I realize the importance of taking time to really examine a concept instead of dishing out a ditto with a bunch of problems. The importance of knowing how and why to do math rather than just answering a question has become even more apparent. (Chris)

> I feel it is important for the teacher to provide a model that will stimulate children to think about varied ways of problem solving. Students need opportunities to think for themselves. (Kristin)

> I like the way she [instructor] illustrates with cubes and blocks. By playing around with things, letting us do things with groups, we come to a solution rather than just having us sit down and work with math books. Maybe we should introduce problems like she did in our class. By letting kids build up things, they'll see things clearer than having them spend 20 minutes working in their math book quietly or just having the teacher at the black board. They will have a better understanding. It will make math easier. I think they need something to represent, something concrete that they can see. I was not aware before of patterns. I was used to being given a formula. You have a better understanding when you figure it out for yourself. In the long run it is easier to remember. (Kelly)

The students during the 10-week course investigated a number of problem situations. That experience resulted in a recognition of and appreciation for patterns in mathematics, an ability to begin to see relationships among mathematical ideas, and a sense that the teaching and learning of mathematics could be enhanced by posing problem situations for their students to puzzle over. But a significant number continued to believe that at the elementary level problem solving should be pursued only after basics had been mastered. Problem solving was viewed as a separate topic rather than a topic integrated with skill development.

Several items in the final questionnaire were designed to assess student beliefs about the relationship between problem solving and skill development in elementary mathematics. More than half expressed the belief that when students were having difficulties with a topic the teacher should zero in on skill deficiencies and provide additional drill and practice. Half indicated

that children needed to master computational skills before going on to problem solving. More than half agreed with the statement that it is important to master facts and skills at one level before going on because each level builds on previous ones.

Opportunities for group work. In most mathematics classrooms, students seldom have opportunities to work together with classmates on problem solving. Learning mathematics is an individual effort. This practice is sustained in part by notions that when students work in groups considerable classroom management problems emerge, that some students will let others do all the work and copy answers, and that teachers cannot effectively monitor and evaluate individual student learning. In this class, students were encouraged to work on problem situations in groups.

Initially, more than half of the students were not sure about the advisability of group work. As Kelly put it,

> Group work allows you to get ideas from others when you are unsure of where you are going. But some students may take the easy way out, let others in the group do the work or they won't speak up when they have a question. They got the answer but they don't understand.

By the second interview, she was enthusiastic about group work. Her belief that some might "take the easy way out" had been challenged by her actual experience in the class:

> I like being more active, involved, where we can work things out rather than having the instructors demonstrate everything on the board. This is the most comfortable class I have ever had. When she [instructor] ends a class with a further problem for us to consider, she wants to leave us with a question in our head. "Your group helped, class helped, now it's time to do some thinking on your own."

Students reported that working in groups to solve math problems was a good way for them to see alternative strategies for thinking about problems.

> Chris: I think it's really good. It's one of the best parts of the class. Because with these problems you can sit

at home forever and not get any further but just one thing from someone else--it may seem pretty trivial to them--but you think, oh yeah, that's right.

Interviewer: Do you think your way of participating in the group has changed at all?

Chris: Maybe some. I think people now are more apt to throw out suggestions. At first, people said "I don't know." Now we're more free to try anything and now people say, "OK, let's see if this happens."

Interviewer: Why do you think that is?

Chris: Maybe we've learned that's what you kinda have to do, sometimes just start making guesses and see what you get. Also, people are more relaxed in groups. They know each other. They don't care if you kinda screw up.

Maria: [Group work is important] because if one student doesn't understand, maybe another can explain better than the teacher. I like working together, it's easier for me tounderstand if someone can explain differently from the teacher. I think it is important for kids to work together.

From a tentative beginning, students working in small groups began to develop a sense of collective responsibility for their learning. In part, this was facilitated by the instructor's admonition that she would not respond to a student's question unless it was a question shared by all group members. This strategy promoted questioning within the group, reliance on each other to puzzle over the problem situation, and a shared responsibility to not consider a problem solved until there was total understanding among all group members.

Maria: My group got the [staircase] problem just like that. I said, "Wait a minute, wait a minute. Explain that." I didn't see it. . . . Another person tried to explain and realized she didn't understand either. Then one person explained and with the combination of the two of them, then I understood.

Interviewer: I noticed you wouldn't let the people in your group go on until you were clear about something. By being persistent, you found out that not everyone in the group really understood.

Maria: Yeah!

Chris: I really like a lot what we do in groups. We discuss it, work it out together . . . . A lot of time is spent talking about problems, not just doing them on your own. It is important really, discussing, thinking. It changes attitudes and confidence.

Kelly was conscious of the role she played in her group:

> I have a hard time explaining to other people. That's why I like group work. It's easier to explain than to the whole class. . . . This is the kind of discussion class which you hardly ever see in math. It's nice to hear someone also confused and it's nice to know you can help someone. If I understand, I don't want to do all the talking. I'll give a little input, maybe I'll ask a question that I think will help. Sometimes going over an explanation, I get things *I* missed before. At first look, I might have no idea of what the result will be. These larger problems are a struggle. But in groups, see how we put things together! You work with each other until you understand. And we come up with the solution. I've learned from the group things I wouldn't have thought of.

Group work provided opportunities for students to share different strategies for problem solving and to puzzle together over problem situations. In addition, group work provided a context for students to develop confidence in their own abilities and to trust their own thinking. A consistent assessment by the students we interviewed was that group work promoted exploration and experimentation, new ways of thinking about mathematics, different ways of looking at problem situations, and, ultimately, better understanding.

While students acknowledged the benefit of group work for their own learning, as prospective teachers, a few expressed reservation about the use of group work in their future classrooms. For example Kristin said,

> Group work would be nice but time wise there are a lot of restrictions. The district tells you what to do. Young kids need to learn basic arithmetic, addition, subtraction, multiplication, and division. Probably I'll be using a lot of ditto sheets. I'm sure I could use all these things that we've done in class but it seems like the time restrictions and district requirements say that you have to get through multiplication and stuff first.

<u>Opportunities to talk about mathematics</u>. The typical mathematics class offers limited opportunities to communicate about mathematics. Developing

students' ability to talk about the mathematics they are learning, to describe how they approached a problem and to justify a solution is part of empowering them with mathematics. Group problem solving provides experiences for students to develop a common mathematical language. It provides opportunities for students to think about how one determines or "knows" whether a solution is "right" and how one justifies solutions to members of the small group. And it shifts authority for validating conjectures and solutions from the teacher to the students and their own mathematical reasoning.

At the beginning of the class, the majority of students slightly agreed with this questionnaire statement: "As they do mathematics, students should have frequent opportunities to discuss their ideas with classmates and get their classmates' responses." When they completed the questionnaire at the conclusion of the course, all strongly agreed with the statement. In initial interviews, all the students spoke of their discomfort and resistance to talking about mathematics "because it's difficult." At the same time, they seemed to acknowledge that talking about math could be useful.

> Melanie: Well, I'm not really comfortable with speaking my "whys" or just telling why I did what I did. I know that if I could do it, it would make things a lot easier as far as learning math and teaching it. Sometimes I can't express it in words. It's almost like a mental block. But I'm trying.

Kelly mentioned frequently the importance of vocabulary "if you want to understand." A recurring theme throughout was her sense of the importance of "putting into words," "trying to explain things," "finding the exact words to say what I want to say." She found the first writing assignment difficult because "I knew what they were saying but I had a hard time trying to explain it." Even as her work in groups provided confirmation of her ability to help her classmates, she worried, "I think I can't explain well."

Chris also expressed both frustration with and recognition of the importance of talking about mathematics:

> It's good but it's annoying. When you're first asked, it's awful. You feel like "I don't know how I got it, I just did it." It does help when you're *forced* to say why you did that. It helps to hear someone else explain their answer. You say, "That makes sense," rather than just giving the answers. When she [instructor] asks you a question like "Why did you do that," it forces you to look at *why* you did it. Just by verbalizing it, it becomes clear. . . . But by actually saying it, it's kind of like looking at what you're doing. When other people do it, you don't know what they're thinking but by them saying it, I think, "Oh yeah, that *is* a good idea."

In the final interview, Chris elaborated,

> Sometimes math concepts can be kind of abstract and just to do them on paper helps because you can see it. But just as important is talking about it and thinking it through. You can like sit there and do a problem and really not understand it. Making you talk about it really helps work things out or even if you hear other people say stuff it can really clear up a problem for you.

This course provided frequent opportunities for students to communicate mathematically, including talking, listening, reading, writing, and demonstrating. The students were beginning to see the usefulness of having multiple ways to communicate ideas or approaches to problem situations. And they found that listening to the ideas of others sometimes caused them to modify their own ideas or helped to clarify partially understood ideas or processes.

## Summary and Conclusions

A 10-week course cannot completely transform the traditional view of mathematics teaching and learning that prospective elementary teachers bring to it. This course did succeed in challenging intending teachers' notions about what it means to know mathematics. As learners of mathematics they were beginning to develop a conceptual understanding of long familiar mathematical ideas and they were beginning to value a learning environment organized around problem solving, group work, and opportunities to talk about mathematics. But

they continued to hold many of the traditional notions that they brought to the course about teaching and learning mathematics at the elementary level.

At the beginning of the course, virtually all students talked about mathematics as "the basics"--adding, subtracting, multiplying, and dividing. Knowing mathematics meant being able to recall appropriate procedures or formulas. Doing mathematics called for plugging numbers into equations. By the end of the course, they talked differently about their own learning of mathematics. They talked with amazement about the recurrence of patterns in the mathematical ideas they had explored in class. They spoke of the connections between ideas that they had never encountered before. Many for the first time came to understand why rules and procedures they had memorized years ago really worked.

Students came to this class expecting the typical routine--teacher lecture and demonstration, homework assignments, and tests. What they encountered did not resemble this routine at all. And the experience led them to question the effectiveness of the "typical routine." The exploration of problem situations enabled them to uncover patterns and see relationships among mathematical ideas. They welcomed group work because it allowed them to puzzle together over problem situations and to see the different strategies people used to solve a problem. While some were reluctant to ask questions in the large group, they felt more at ease to question or to make tentative suggestions in the small group. They accepted responsibility for each other's learning and evidenced a willingness to persist when solutions were not immediately apparent. The frequent opportunities to communicate mathematically resulted in an appreciation for the importance of the use of language, both natural and mathematical, in the classroom.

Several factors contributed to these changes: (a) the framework that organized the course with an emphasis on patterns, relationships among mathematical ideas, and multiple representations of mathematical concepts; (b) the introduction of new topics in the course with an interesting and challenging problem situation; (c) the richness of the mathematics embedded in each problem situation; (d) an environment where students could cooperatively learn ways to make sense of mathematics, invent strategies to solve new problems, and build models to understand mathematical concepts.

But at least half persisted with their belief that group work and frequent opportunities to communicate mathematically in the elementary classroom were not realistic. Time and organizational constraints would limit their ability to implement instructional practices that they had found so useful for themselves. They believed they would be responsible for teaching a basic, traditional curriculum. Their limited knowledge of the elementary curriculum and their expectations of young learners were not significantly challenged or altered. As Kelly put it:

> I'll be teaching times tables and I'll use her [instructor] approach for understanding. Teach simple patterns, two blocks, four blocks, and through that introduce the 2s times tables. They're not really getting into story problems. They begin just to learn basics they'll use throughout their lives.

Efforts to improve the teaching and learning of mathematics through conceptually based instruction are frustrated, in part, by a belief that arithmetic should constitute the curriculum. It is a belief shared by teachers, administrators, and parents. Teachers have tended not to have sufficient subject matter knowledge to challenge these notions. The course piloted and reported on here is the first of a three-term sequence. A single 10-week course is clearly insufficient to equip teachers to resist the contextual constraints that impede the implementation of conceptual

approaches. Yet the results of this pilot suggest important changes did occur in student thinking about teaching and learning mathematics for themselves. A yearlong conceptually based mathematics course and an integrated methods course may come closer to providing prospective elementary teachers with the knowledge necessary to promote a different vision of elementary mathematics teaching. Over the next two years the researchers will study the implementation of this sequence of courses as they we follow a cohort of students through the courses, student teaching, and their first year in the classroom.

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